Extensive Form Games

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We will look at another model for games, called *extensive form games*. We will then show how it can be reduced to a strategic form game.

An extensive form game captures games where players move sequentially. Such a game is usually visualized as a tree.

An extensive form game Γ is represented as the tuple $(N, (A_i)_{i \in N}, H, P, (I_i)_{i \in N}, (u_i)_{i \in N})$ where

- N is the set of players.
- A_i is the set of actions available to player i.
- A history is a sequence of actions that can be played during the game. The state of the game can be represented by the history at that point in time.
- H is the set of terminal histories, i.e., when the game reaches a state in H, the game ends. No history in H is a prefix of another history in H.
- S_H is the set of proper subhistories of H. Formally, let prefix(h) denote the set of proper prefixes of history $h \in H$. Then $S_H := \bigcup_{h \in H} \text{prefix}(h)$.
- $P : S_H \mapsto N$ is the player function, i.e., when the game is at state $h \in S_H$, the player $P(h)$ is supposed to make a move.
- For $i \in N$, let H_i be the states where i is supposed to make a move, i.e., $H_i := \{h \in S_H : P(h) = i\}$. Then I_i is a partition of H_i. Sets in I_i are called the information sets of player i. Intuitively, for each $X \in I_i$, player i cannot distinguish the states in X.
- For each $i \in N$, $u_i : H \mapsto \mathbb{R}$ is the utility function of player i. This means that when the game reaches state $h \in H$, each player i will get utility $u_i(h)$.
- For history $h \in S_H$, where $P(h) = i$, let $C(h) \subseteq A_i$ be the actions available to player i, i.e., $a \in C(h) \iff h + a \in S_H \cup H$. For each $i \in N$ and each $X \in I_i$, $C(h)$ should be the same for each $h \in X$. Intuitively, the actions available to player i should be the same for all histories in an information set. Denote these actions as $C(X)$.

Definition 1. *A game is said to be a perfect information game iff for each player i, all information sets are singletons.*

Definition 2. *For a player i, $s_i : I_i \mapsto A_i$ is called a* strategy *iff for each $J \in I_i$, we have $s_i(J) \in C(J)$.*

Intuitively, a strategy is a plan about which action to take for each information set.

Definition 3. *Let s_i be a strategy of player i. Let $s := (s_i)_{i \in N}$. Then s is called a* strategy profile *for the game. Let S_i be all possible strategies for player i. Let $S := S_1 \times S_2 \times \ldots \times S_n$, where $n := |N|$. Then S is called the* strategy profile collection *for the game.*

For a strategy profile s, the outcome of the game, denoted by $O(s)$, is the terminal history reached when the game ends. For a player i and strategy profile s, $u_i(s) := u_i(O(s))$.

The strategic form equivalent *of Γ is the game $(N, (S_i)_{i \in N}, (u_i)_{i \in N})$.*