New Fairness Concepts for Allocating Indivisible Items

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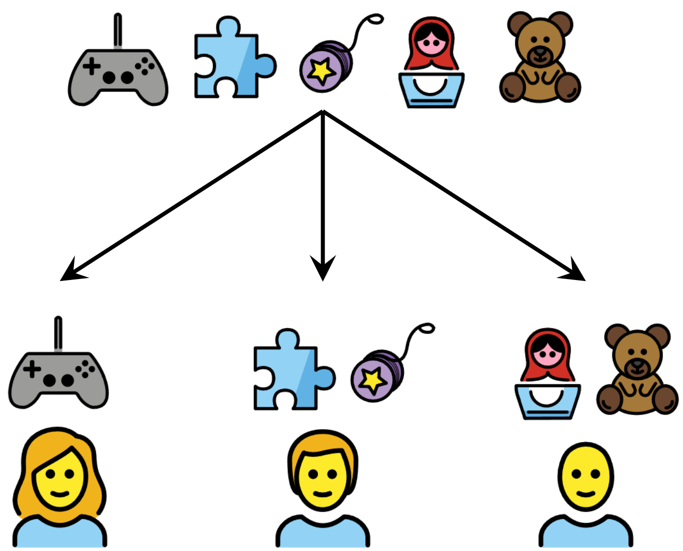
Fair Division of Goods

Divide goods among n people (called agents), who are all ‘equally deserving’.



Formalizing the Problem

- Set $N = \{1, 2, \ldots, n\}$ of agents.
- Set $M = \{1, 2, \ldots, m\}$ of goods.
- $v_i(g) \in \mathbb{R}_{\geq 0}$ is called i's valuation for good $g \in M$.

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- $v_i(g) \in \mathbb{R}_{\geq 0}$ is called i's valuation for good $g \in M$.
- v_i is called agent i's valuation function.
- Extending to subsets of goods:
  - For $S \subseteq M$, $v_i(S) = \sum_{g \in S} v_i(g)$.

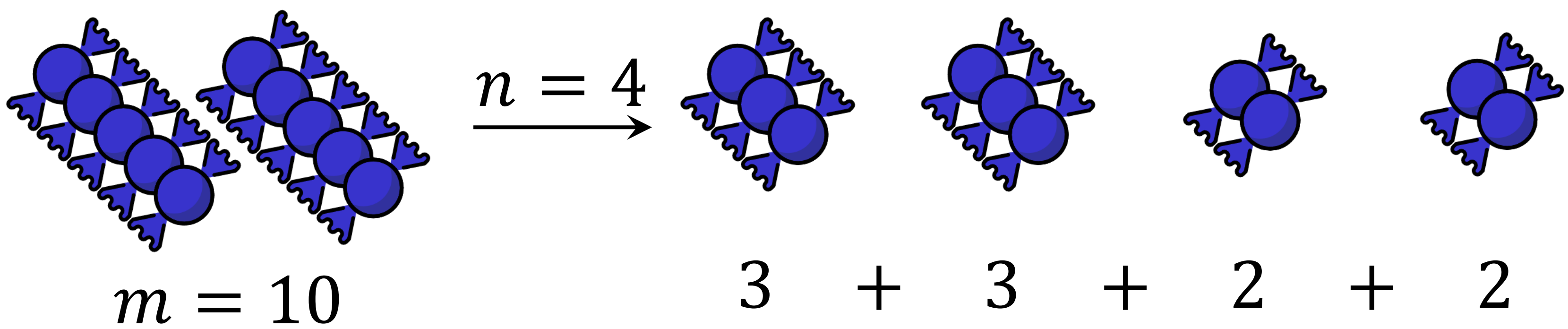
Formalizing the Problem (cont.)

- An allocation X is a specification of who gets what: an n-tuple $(X_1, X_2, \ldots, X_n)$ where X_i is the set of goods that agent i gets.
- X_i is called agent i's *bundle* in allocation X.
- We need to find an allocation that is ***fair***.

Notions of Fairness

Simple example

- Suppose there are m identical goods and n agents.
- Each agent should get $\lfloor m/n \rfloor$ or $\lceil m/n \rceil$ goods.



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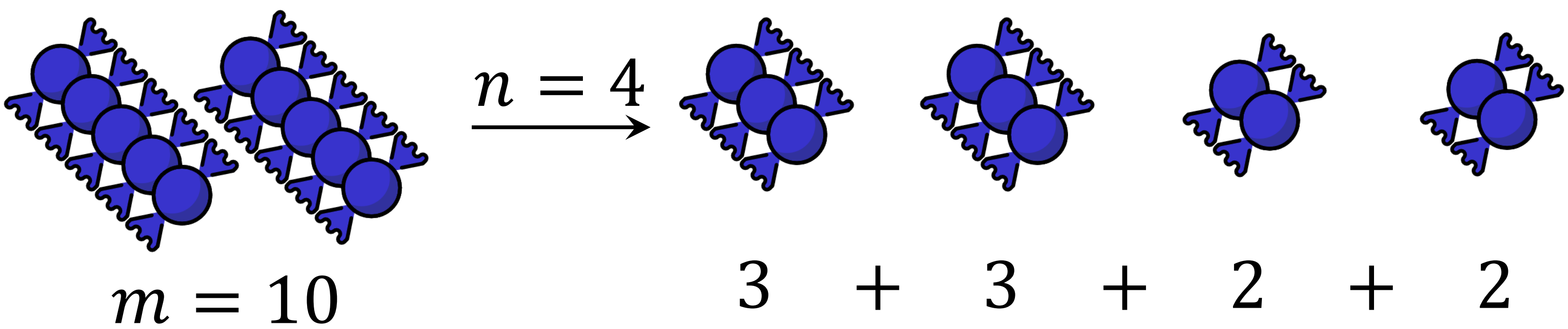

- How do we generalize this idea? 2 observations:
  - $\lceil m/n \rceil - \lfloor m/n \rfloor \leq 1.$
  - Each agent gets roughly $1/n$ fraction of goods.

EF and EFX

- In allocation X,
  - agent i **envies** agent j if $v_i(X_i) < v_i(X_j)$.
  - agent i **strongly envies** agent j if $\exists g \in X_j$ s.t. $v_i(X_i) < v_i(X_j - \{g\})$.
- X is envy-free (**EF**) if no one envies anyone else.
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	🍭	🍪	🍬	🍩
👩	4	4	2	7
👨	14	14	12	17

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Existence of EF and EFX

- EF allocations may not exist (e.g., single good).
- Important problems:
  - Do EFX allocations always exist?
  - Can we efficiently compute EFX allocations?
- EFX exists for special cases ($n \leq 3$ or identical v_i).
- Open problem since 2016.
- Relaxations of EFX have been studied:
  - EF1 [EC'04], α-EFX [TCS'20], EFX-with-charity [SODA'20].

PROP and MMS

- Allocation X is PROP if $\forall i$, $v_i(X_i) \geq v_i(M)/n$.
- PROP allocations may not exist (e.g., single good).
- An allocation X is MMS if for every agent i,
$$v_i(X_i) \geq \max_Z \min_j v_i(Z_j)$$
- MMS had been a compelling fairness notion for a long time, but in 2014 it was shown to not always exist.

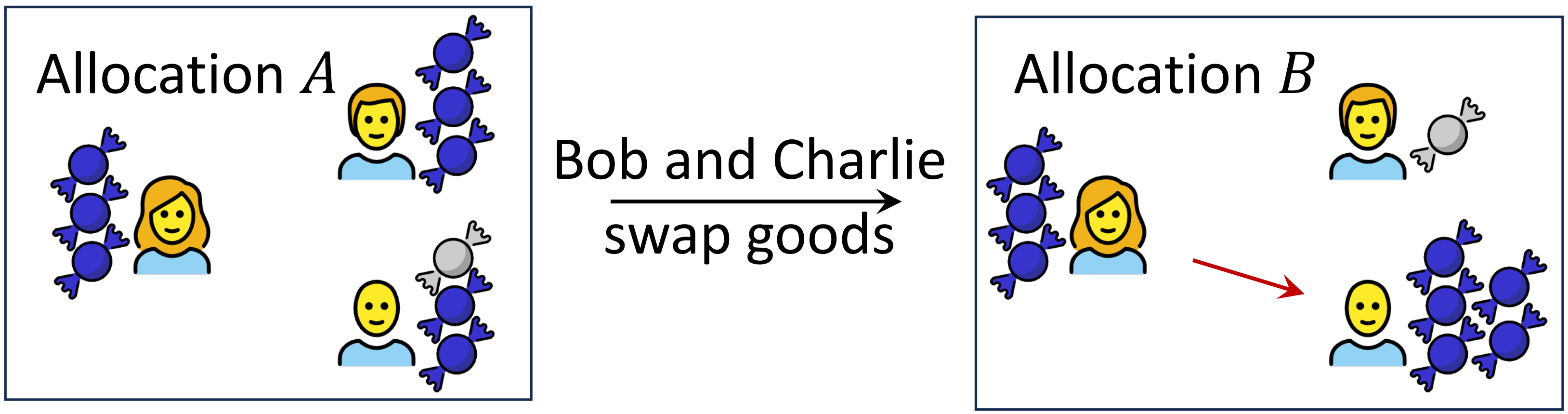
Towards a different notion of fairness

EFX and MMS currently can’t be used.

We show a relaxation of EFX that’s almost as good as EFX.

Motivating Example

- 3 agents (Alice, Bob, Charlie) and 9 goods.
- All goods are identical to Alice.



- Alice has the same bundle in A and B, yet she considers A fair (by EFX) and B unfair.

The MYOB principle

- Mind-Your-Own-Business (MYOB) principle: whether an allocation is fair to you should depend only on your own bundle.
- How the remaining goods are distributed among the other agents is none of your business.

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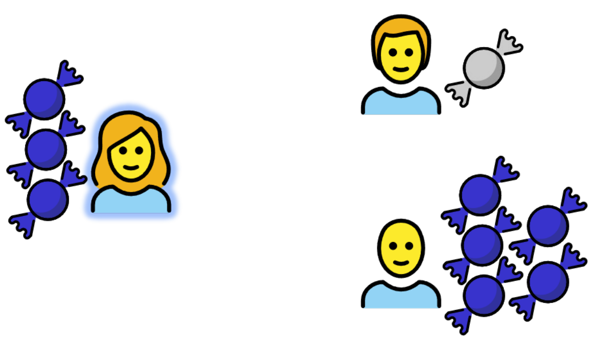
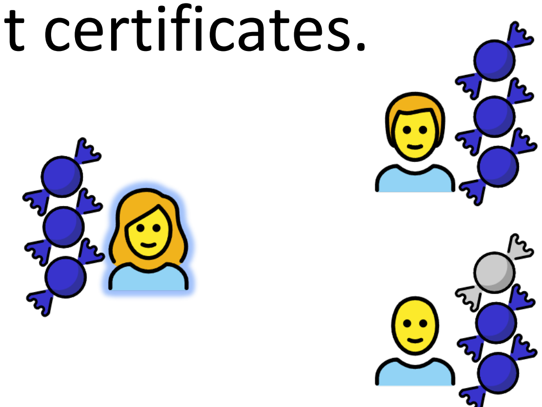
- PROP and MMS follow MYOB. EF and EFX don’t.
- Violating MYOB doesn’t make a fairness notion bad.
- EFX is too demanding.

Epistemic fairness

- An allocation X is **epistemic EFX** if for each agent i,
  - we can redistribute goods outside X_i to agents $N \setminus \{i\}$
  - such that i doesn't strongly envy anyone anymore.
- Formally, allocation X is Epistemic EFX if for each agent i, there is an allocation Y s.t. $X_i = Y_i$ and agent i is strong-envy-free in Y.
- Y is called i's certificate of fairness. Different agents can have different certificates.



X (not EFX, but Epistemic EFX for Alice)



Y (EFX for Alice)

Epistemic EFX

- Epistemic EFX follows MYOB.
- Although Epistemic EFX is a relaxation of EFX, it seems to be almost as good as EFX.
- Do Epistemic EFX allocations always exist? Yes!

Our Contributions

- [BK EC'17] gave a polytime algorithm for 2/3-MMS. We show that their algorithm's output is also Epistemic EFX.
- MMS $\implies$ Epistemic EFX $\implies$ PROP1.

Open Problems

1. Epistemic EFX for non-additive valuations.
2. Epistemic EFX + other notions of fairness:
    - EF1, α-EFX, α-MMS.
3. Epistemic EFX + PO.

Thank You

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Algorithm

- An instance is ordered if for each agent i, $v_i(1) \geq v_i(2) \geq \cdots \geq v_i(m)$.

	6	5	1	1	2
	3	2	10	11	1
	6	6	1	2	1

I

→

	g_1	g_2	g_3	g_4	g_5
	6	5	2	1	1
	11	10	3	2	1
	6	6	2	1	1

$\hat{I} = \text{ordered}(I)$

↓

EFX allocation $\hat{A}$ for $\hat{I}$

←

Epistemic EFX allocation A for I